Centre Number
Student Number

NSW GOVERNMENT

NSW Education Standards Authority

2023 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–18)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is equal to $(a + ib)^3$?

A. $\left(a^3 - 3ab^2\right) + i\left(3a^2b + b^3\right)$

B. $\left(a^3 + 3ab^2\right) + i\left(3a^2b + b^3\right)$

C. $\left(a^3 - 3ab^2\right) + i\left(3a^2b - b^3\right)$

D. $\left(a^3 + 3ab^2\right) + i\left(3a^2b - b^3\right)$

2 Consider the following statement.

‘If an animal is a herbivore, then it does not eat meat.’

Which of the following is the converse of this statement?

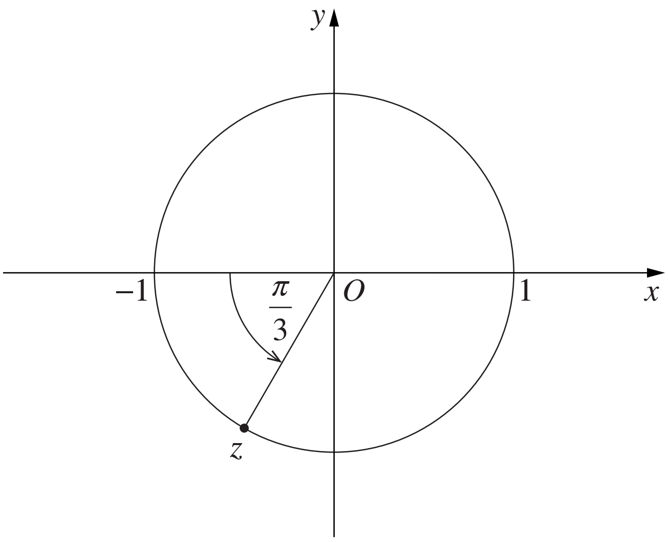
A. If an animal is a herbivore, then it eats meat.

B. If an animal is not a herbivore, then it eats meat.

C. If an animal eats meat, then it is not a herbivore.

D. If an animal does not eat meat, then it is a herbivore.

3 A complex number z lies on the unit circle in the complex plane, as shown in the diagram.



Which of the following complex numbers is equal to $\overline{z}$?

A. $-z$

B. z^2

C. $-z^3$

D. z^4

4 Consider the following statement about real numbers.

> 'Whichever positive number r you pick, it is possible to find a number x greater than 1 such that $\dfrac{\ln x}{x^3} < r$.'

When this statement is written in the formal language of proof, which of the following is obtained?

A. $\forall\, x > 1 \quad \exists\, r > 0 \quad \dfrac{\ln x}{x^3} < r$

B. $\exists\, x > 1 \quad \forall\, r > 0 \quad \dfrac{\ln x}{x^3} < r$

C. $\forall\, r > 0 \quad \exists\, x > 1 \quad \dfrac{\ln x}{x^3} < r$

D. $\exists\, r > 0 \quad \forall\, x > 1 \quad \dfrac{\ln x}{x^3} < r$

5 Which of the following is a true statement about the lines $\ell_1 = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ and $\ell_2 = \begin{pmatrix} 3 \\ -10 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$?

A. ℓ_1 and ℓ_2 are the same line.

B. ℓ_1 and ℓ_2 are not parallel and they intersect.

C. ℓ_1 and ℓ_2 are parallel and they do not intersect.

D. ℓ_1 and ℓ_2 are not parallel and they do not intersect.

6 Which of the following functions does NOT describe simple harmonic motion?

A. $x = \cos^2 t - \sin 2t$

B. $x = \sin 4t + 4\cos 2t$

C. $x = 2\sin 3t - 4\cos 3t + 5$

D. $x = 4\cos\left(2t + \frac{\pi}{2}\right) + 5\sin\left(2t - \frac{\pi}{4}\right)$

7 Which of the following statements about complex numbers is true?

A. For all real numbers x, y, θ with $x \neq 0$,

$$\tan\theta = \frac{y}{x} \quad \Rightarrow \quad x + iy = re^{i\theta},$$

for some real number r.

B. For all non-zero complex numbers z_1 and z_2,

$$\mathrm{Arg}(z_1) = \theta_1 \text{ and } \mathrm{Arg}(z_2) = \theta_2 \quad \Rightarrow \quad \mathrm{Arg}(z_1 z_2) = \theta_1 + \theta_2,$$

where Arg denotes the principal argument.

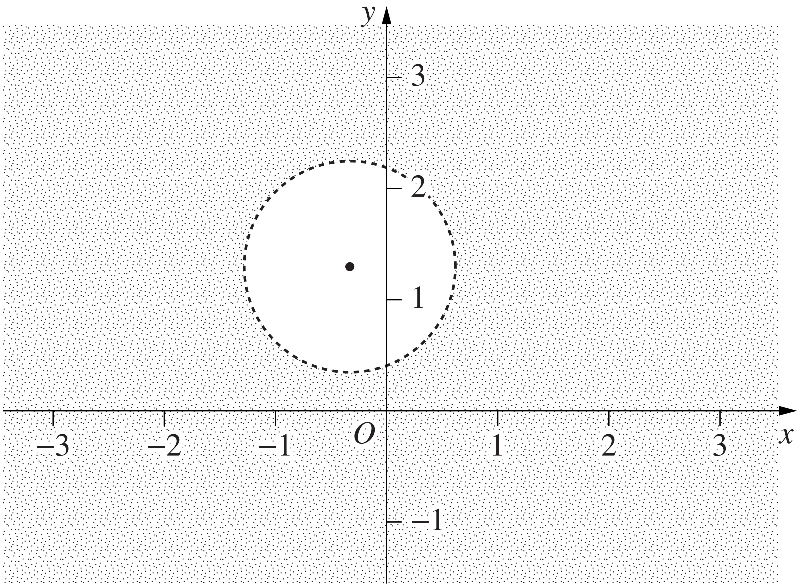
C. For all real numbers $r_1, r_2, \theta_1, \theta_2$ with $r_1, r_2 > 0$,

$$r_1 e^{i\theta_1} = r_2 e^{i\theta_2} \quad \Rightarrow \quad r_1 = r_2 \text{ and } \theta_1 = \theta_2.$$

D. For all real numbers x, y, r, θ with $r > 0$ and $x \neq 0$,

$$x + iy = re^{i\theta} \quad \Rightarrow \quad \theta = \arctan\left(\frac{y}{x}\right).$$

8 A shaded region on a complex plane is shown.



Which relation best describes the region shaded on the complex plane?

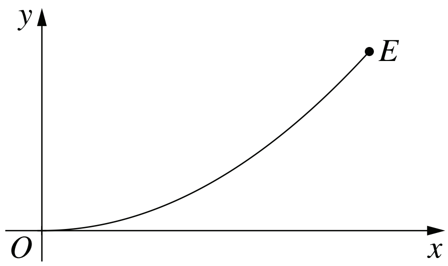
A. $|z - i| > 2|z - 1|$

B. $|z - i| < 2|z - 1|$

C. $|z - 1| > 2|z - i|$

D. $|z - 1| < 2|z - i|$

9 A particle travels along a curve from O to E in the xy-plane, as shown in the diagram.



The position vector of the particle is $\boldsymbol{r}$, its velocity is $\boldsymbol{v}$, and its acceleration is $\boldsymbol{a}$.

While travelling from O to E, the particle is always slowing down.

Which of the following is consistent with the motion of the particle?

A. $\boldsymbol{r} \cdot \boldsymbol{v} \leq 0$ and $\boldsymbol{a} \cdot \boldsymbol{v} \geq 0$

B. $\boldsymbol{r} \cdot \boldsymbol{v} \leq 0$ and $\boldsymbol{a} \cdot \boldsymbol{v} \leq 0$

C. $\boldsymbol{r} \cdot \boldsymbol{v} \geq 0$ and $\boldsymbol{a} \cdot \boldsymbol{v} \geq 0$

D. $\boldsymbol{r} \cdot \boldsymbol{v} \geq 0$ and $\boldsymbol{a} \cdot \boldsymbol{v} \leq 0$

10 Consider any three-dimensional vectors $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and $\underset{\sim}{c} = \overrightarrow{OC}$ that satisfy these three conditions

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1$$
$$\underset{\sim}{b} \cdot \underset{\sim}{c} = 2$$
$$\underset{\sim}{c} \cdot \underset{\sim}{a} = 3.$$

Which of the following statements about the vectors is true?

A. Two of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ could be unit vectors.

B. The points A, B and C could lie on a sphere centred at O.

C. For any three-dimensional vector $\underset{\sim}{a}$, vectors $\underset{\sim}{b}$ and $\underset{\sim}{c}$ can be found so that $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ satisfy these three conditions.

D. $\forall$ $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ satisfying the conditions, $\exists$ r, s and t such that r, s and t are positive real numbers and $r\underset{\sim}{a} + s\underset{\sim}{b} + t\underset{\sim}{c} = \underset{\sim}{0}$.

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Solve the quadratic equation **2**

$$z^2 - 3z + 4 = 0,$$

where z is a complex number. Give your answers in Cartesian form.

(b) Find the angle between the vectors **3**

$$\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$$
$$\underset{\sim}{b} = -\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k},$$

giving your answer to the nearest degree.

(c) Find a vector equation of the line through the points $A(-3, 1, 5)$ and $B(0, 2, 3)$. **2**

(d) The quadrilaterals $ABCD$ and $ABEF$ are parallelograms. **2**

By considering $\overrightarrow{AB}$, show that $CDFE$ is also a parallelogram.

(e) A particle moves in simple harmonic motion described by the equation **2**

$$\ddot{x} = -9(x - 4).$$

Find the period and the central point of motion.

(f) Find $\displaystyle\int_0^2 \frac{5x - 3}{(x + 1)(x - 3)}\,dx$. **4**

Question 12 (15 marks) Use the Question 12 Writing Booklet

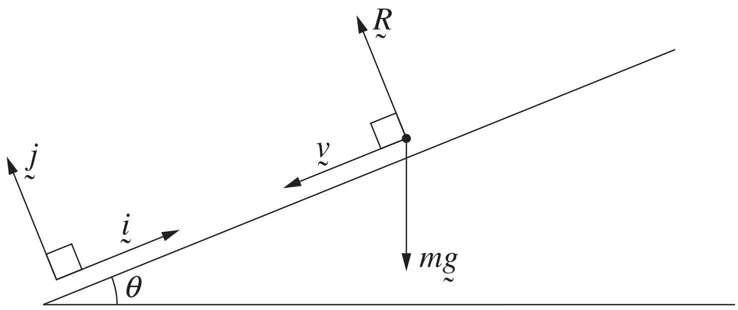
(a) Prove that $\sqrt{23}$ is irrational. **3**

(b) Prove that for all real numbers x and y, where $x^2 + y^2 \neq 0$, **2**

$$\frac{(x+y)^2}{x^2+y^2} \leq 2.$$

(c) An object with mass m kilograms slides down a smooth inclined plane with velocity $\underset{\sim}{v}(t)$, where t is the time in seconds after the object started sliding down the plane. The inclined plane makes an angle θ with the horizontal, as shown in the diagram. The normal reaction force is $\underset{\sim}{R}$. The acceleration due to gravity is $\underset{\sim}{g}$ and has magnitude g. No other forces act on the object.

The vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors parallel and perpendicular, respectively, to the plane, as shown in the diagram.



(i) Show that the resultant force on the object is $\underset{\sim}{F} = -(mg\sin\theta)\underset{\sim}{i}$. **2**

(ii) Given that the object is initially at rest, find its velocity $\underset{\sim}{v}(t)$ in terms of g, θ, t and $\underset{\sim}{i}$. **2**

Question 12 continues on page 10

(d) Find the cube roots of $2 - 2i$. Give your answer in exponential form. **3**

(e) The complex number $2 + i$ is a zero of the polynomial

$$P(z) = z^4 - 3z^3 + cz^2 + dz - 30$$

where c and d are real numbers.

(i) Explain why $2 - i$ is also a zero of the polynomial $P(z)$. **1**

(ii) Find the remaining zeros of the polynomial $P(z)$. **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) Find $\displaystyle\int \frac{1-x}{\sqrt{5-4x-x^2}}\,dx$. **3**

(b) (i) Show that $k^2 - 2k - 3 \geq 0$ for $k \geq 3$. **1**

(ii) Hence, or otherwise, use mathematical induction to prove that $2^n \geq n^2 - 2$, for all integers $n \geq 3$. **3**

Question 13 continues on page 12

(c) A particle of mass 1 kg is projected from the origin with speed 40 m s^{-1} at an angle 30° to the horizontal plane.

(i) Use the information above to show that the initial velocity of the particle is $\mathbf{v}(0) = \begin{pmatrix} 20\sqrt{3} \\ 20 \end{pmatrix}$. **1**

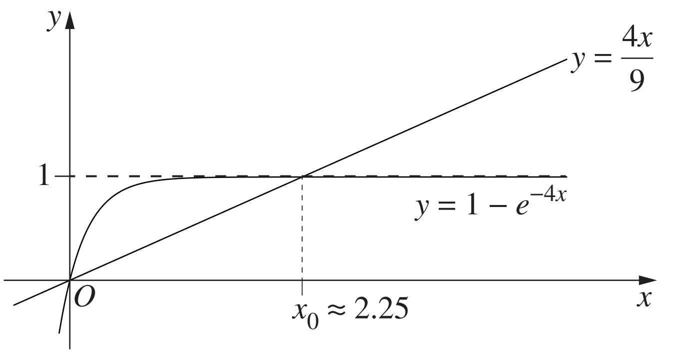
The forces acting on the particle are gravity and air resistance. The air resistance is proportional to the velocity vector with a constant of proportionality 4. Let the acceleration due to gravity be 10 m s^{-2}.

The position vector of the particle, at time t seconds after the particle is projected, is $\mathbf{r}(t)$ and the velocity vector is $\mathbf{v}(t)$.

(ii) Show that $\mathbf{v}(t) = \begin{pmatrix} 20\sqrt{3}e^{-4t} \\ \frac{45}{2}e^{-4t} - \frac{5}{2} \end{pmatrix}$. **3**

(iii) Show that $\mathbf{r}(t) = \begin{pmatrix} 5\sqrt{3}\left(1 - e^{-4t}\right) \\ \frac{45}{8}\left(1 - e^{-4t}\right) - \frac{5}{2}t \end{pmatrix}$. **2**

(iv) The graphs $y = 1 - e^{-4x}$ and $y = \dfrac{4x}{9}$ are given in the diagram below. **2**



Using the diagram, find the horizontal range of the particle, giving your answer rounded to one decimal place.

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Let z be the complex number $z = e^{\frac{i\pi}{6}}$ and w be the complex number $w = e^{\frac{3i\pi}{4}}$.

(i) By first writing z and w in Cartesian form, or otherwise, show that $|z+w|^2 = \dfrac{4-\sqrt{6}+\sqrt{2}}{2}$. **3**

(ii) The complex numbers z, w and $z+w$ are represented in the complex plane by the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively, where O is the origin. **2**

Show that $\angle AOC = \dfrac{7\pi}{24}$.

(iii) Deduce that $\cos\dfrac{7\pi}{24} = \dfrac{\sqrt{8-2\sqrt{6}+2\sqrt{2}}}{4}$. **1**

(b) The point P is 4 metres to the right of the origin O on a straight line. **3**

A particle is released from rest at P and moves along the straight line in simple harmonic motion about O, with period 8π seconds.

After 2π seconds, another particle is released from rest at P and also moves along this straight line in simple harmonic motion about O, with period 8π seconds.

Find when and where the two particles first collide.

(c) A projectile of mass M kg is launched vertically upwards from the origin with an initial speed v_0 m s^{-1}. The acceleration due to gravity is g m s^{-2}.

The projectile experiences a resistive force of magnitude kMv^2 newtons, where k is a positive constant and v is the speed of the projectile at time t seconds.

(i) The maximum height reached by the particle is H metres. **3**

Show that $H = \dfrac{1}{2k}\ln\left(\dfrac{kv_0^{\,2}+g}{g}\right)$.

(ii) When the projectile lands on the ground, its speed is v_1 m s^{-1}, where v_1 is less than the magnitude of the terminal velocity. **3**

Show that $g\left(v_0^{\,2}-v_1^{\,2}\right) = kv_0^{\,2}v_1^{\,2}$.

Question 15 (16 marks) Use the Question 15 Writing Booklet

(a) (i) Let $J_n = \int_0^{\frac{\pi}{2}} \sin^n\theta \, d\theta$ where $n \geq 0$ is an integer. **3**

Show that $J_n = \frac{n-1}{n} J_{n-2}$ for all integers $n \geq 2$.

(ii) Let $I_n = \int_0^1 x^n(1-x)^n \, dx$ where n is a positive integer. **4**

By using the substitution $x = \sin^2\theta$, or otherwise,

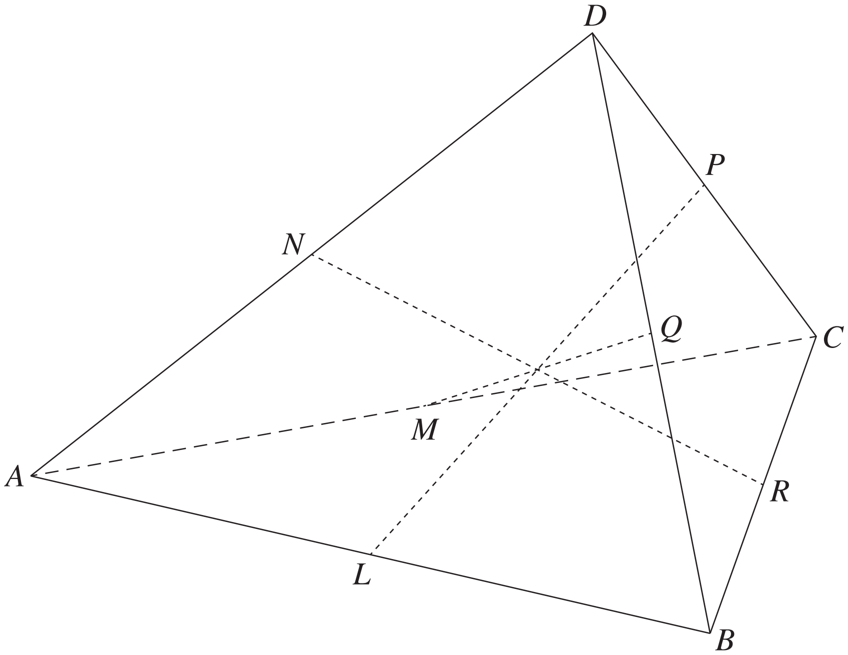
show that $I_n = \frac{1}{2^{2n}} \int_0^{\frac{\pi}{2}} \sin^{2n+1}\theta \, d\theta$.

(iii) Hence, or otherwise, show that $I_n = \frac{n}{4n+2} I_{n-1}$, for all integers $n \geq 1$. **2**

Question 15 continues on page 15

Question 15 (continued)

(b) On the triangular pyramid $ABCD$, L is the midpoint of AB, M is the midpoint of AC, N is the midpoint of AD, P is the midpoint of CD, Q is the midpoint of BD and R is the midpoint of BC.



Let $\underset{\sim}{b} = \overrightarrow{AB}$, $\underset{\sim}{c} = \overrightarrow{AC}$ and $\underset{\sim}{d} = \overrightarrow{AD}$.

(i) Show that $\overrightarrow{LP} = \frac{1}{2}\left(-\underset{\sim}{b} + \underset{\sim}{c} + \underset{\sim}{d}\right)$. **1**

(ii) It can be shown that **3**

$$\overrightarrow{MQ} = \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{c} + \underset{\sim}{d}\right)$$

and $$\overrightarrow{NR} = \frac{1}{2}\left(\underset{\sim}{b} + \underset{\sim}{c} - \underset{\sim}{d}\right).$$

(Do NOT prove these.)

Prove that

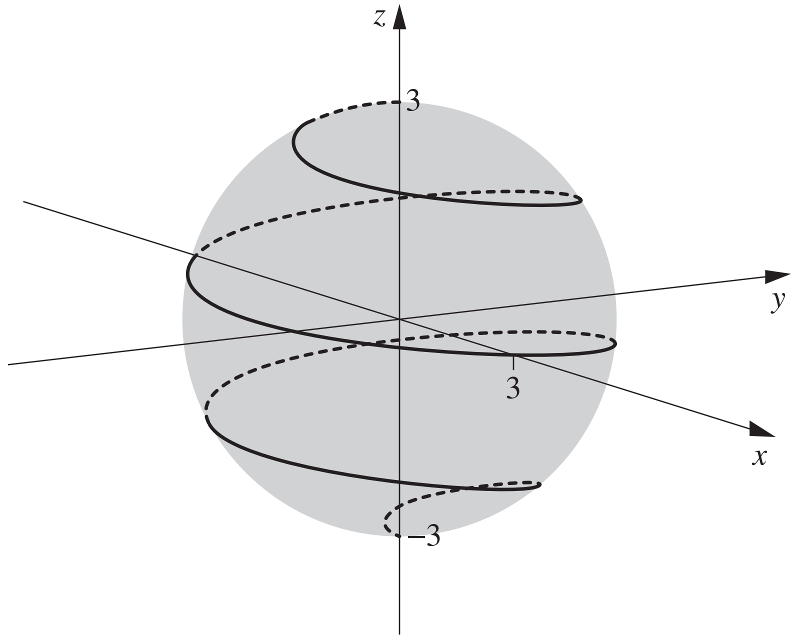
$$\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{AC}\right|^2 + \left|\overrightarrow{AD}\right|^2 + \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{BD}\right|^2 + \left|\overrightarrow{CD}\right|^2 = 4\left(\left|\overrightarrow{LP}\right|^2 + \left|\overrightarrow{MQ}\right|^2 + \left|\overrightarrow{NR}\right|^2\right).$$

Question 15 continues on page 16

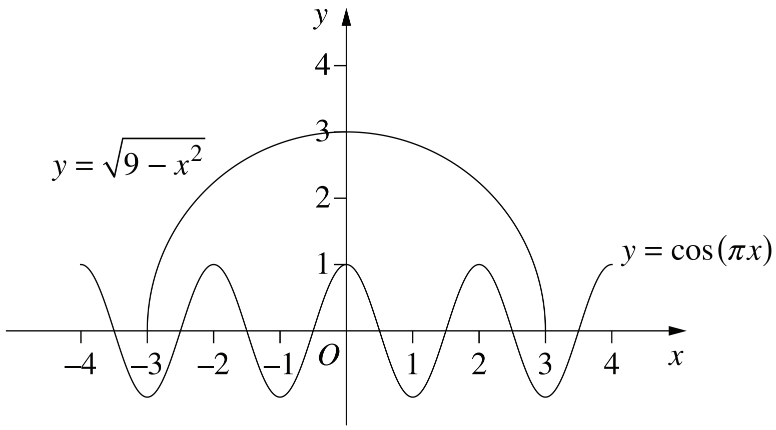
Question 15 (continued)

(c) A curve $\mathscr{C}$ spirals 3 times around the sphere centred at the origin and with radius 3, as shown. **3**

A particle is initially at the point $(0, 0, -3)$ and moves along the curve $\mathscr{C}$ on the surface of the sphere, ending at the point $(0, 0, 3)$.



By using the diagram below, which shows the graphs of the functions $f(x) = \cos(\pi x)$ and $g(x) = \sqrt{9 - x^2}$, and considering the graph $y = f(x)g(x)$, give a possible set of parametric equations that describe the curve $\mathscr{C}$.



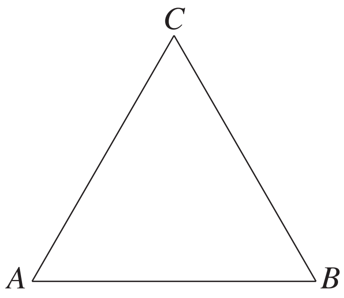
End of Question 15

Question 16 (14 marks) Use the Question 16 Writing Booklet

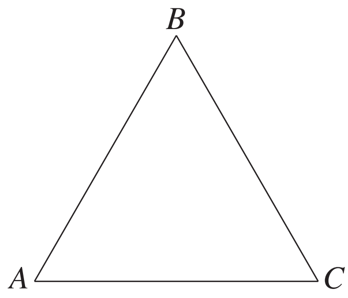
(a) Let w be the complex number $w = e^{\frac{2i\pi}{3}}$.

(i) Show that $1 + w + w^2 = 0$. **2**

The vertices of a triangle can be labelled A, B and C in anticlockwise or clockwise direction, as shown.



ABC is anticlockwise



ABC is clockwise

Three complex numbers a, b and c are represented in the complex plane by points A, B and C respectively.

(ii) Show that if triangle ABC is anticlockwise and equilateral, then $a + bw + cw^2 = 0$. **2**

(iii) It can be shown that if triangle ABC is clockwise and equilateral, then $a + bw^2 + cw = 0$. (Do NOT prove this.) **2**

Show that if ABC is an equilateral triangle, then

$$a^2 + b^2 + c^2 = ab + bc + ca.$$

Question 16 continues on page 18

(b) (i) Prove that $x > \ln x$, for $x > 0$. **2**

(ii) Using part (i), or otherwise, prove that for all positive integers n, **3**

$$e^{n^2+n} > (n!)^2.$$

(c) The complex numbers w and z both have modulus 1, and $\dfrac{\pi}{2} < \operatorname{Arg}\left(\dfrac{z}{w}\right) < \pi$, **3**

where Arg denotes the principal argument.

For real numbers x and y, consider the complex number $\dfrac{xz + yw}{z}$.

On an xy-plane, clearly sketch the region that contains all points (x, y) for which $\dfrac{\pi}{2} < \operatorname{Arg}\left(\dfrac{xz + yw}{z}\right) < \pi$.

End of paper

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2023 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

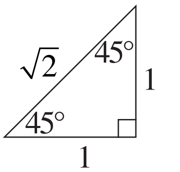
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

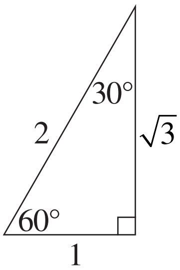
$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

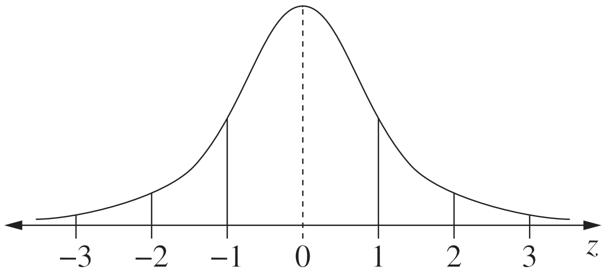
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$